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Numerical determination of Regge poles in potential scattering

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Abstract. Regge poles of local interactions with a $1/r$ singularity at the origin are determined from a polynomial equation which can be solved extremely rapidly on a computer. This equation is found to be particularly suitable for locating the poles at negative energies, but impractical at positive energies. A simple formula is derived for the residue at a pole.

1. Introduction

In nonrelativistic potential scattering Regge poles are generalized bound states and resonances in the complex orbital angular momentum plane (Regge 1960, Moorhouse 1964, Alpharo and Regge 1965, Collins and Squires 1968). A physical bound state occurs whenever a Regge trajectory passes through an integer point along the positive real axis (including the origin) at negative energies whereas a resonance occurs if $\mathrm{Re}\, l$ becomes equal to a positive definite integer (with $|\mathrm{Im}\, l| \ll 1$) at positive energies.

Section 2 contains a summary of relevant results obtained in a previous paper (Warburton and Stern 1969) which forms the basis for the work described in this paper. These earlier results are concerned with the eigenvalues and eigenfunctions of the Lippmann–Schwinger kernel for local interactions $V(r)$ which can be expanded as a power series about $r = 0$, with a $1/r$ singularity at the origin. In § 3 Regge poles of various Yukawa potentials are determined by solving a polynomial equation. This equation is found to be particularly suitable for locating the poles at negative energies. Section 4 contains the derivation of a simple formula for the residue at a pole. A high energy limit for the residue is also deduced. A knowledge of the residues at various Regge poles is useful for evaluating the leading terms in an expansion for the scattering amplitude (Moorhouse 1964, pp 223–58, Aly and Narayanaswamy 1969).

2. Previous results

This section summarizes the relevant results obtained in a previous paper (Warburton and Stern 1969) concerning the eigenvalues $\eta(k, l)$ of the Lippmann–Schwinger kernel, the corresponding wavefunctions $u_l(r)$, and the offshell partial-wave T matrix $T_l(p_1^2, p_2^2; k^2)$ where k^2 is the energy in the centre of mass frame.

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The $\eta(k, l)$ are determined from the partial-wave Schrödinger equation

$$\frac{d^2u_l}{dr^2}+\left(k^2-\frac{l(l+1)}{r^2}-\frac{V(r)}{\eta(k,l)}\right)u_l = 0 \tag{2.1}$$

which is subject to the boundary conditions

$$\begin{aligned} u_l(r) &\sim r^{l+1} \qquad \text{as } r \to 0 \\ u_l(r) &\sim \exp(ikr) \quad \text{as } r \to \infty \end{aligned} \tag{2.2}$$

where we take $\operatorname{Im} k \geqslant 0$, corresponding to the k^2 plane cut along the positive real axis.

By setting

$$K = ik \qquad z = -2Kr \qquad U(r) = -\tfrac{1}{2}rV(r) \tag{2.3}$$

and

$$u_l(r) = r^{l+1}\exp(ikr)\chi(z) \tag{2.4}$$

equation (2.1) is transformed to

$$K\eta\{z\chi''(z)+(2l+2-z)\chi'(z)-(l+1)\chi(z)\} = U\left(\frac{-z}{2K}\right)\chi(z). \tag{2.5}$$

By substituting the expansions

$$\begin{aligned} U(r) &= \sum_{j=0}^{\infty} U_j(-2r)^j \qquad (U_0 \neq 0) \\ \chi(z) &= \sum_{n=0}^{\infty}\sum_{j=0}^{\infty} F_{n,j}z^jK^{-n} \qquad \eta(k,l) = \sum_{j=0}^{\infty}\frac{\alpha_j}{K^{j+1}} \end{aligned} \tag{2.6}$$

into equation (2.5) and equating coefficients of z^jK^{-n} we obtain

$$\sum_{t=0}^{n}[\alpha_t\{(j+1)(j+2l+2)F_{n-t,j+1}-(j+l+1)F_{n-t,j}\}-U_tF_{n-t,j-t}] = 0. \tag{2.7}$$

By applying the condition (Warburton 1966)

$$F_{n,j} = 0 \text{ for } j > n+N-1 \qquad (N = 1, 2, 3, \ldots ; n = 0, 1, 2, \ldots) \tag{2.8}$$

equation (2.7) yields a system of coupled recurrence relations which enable all the $F_{n,j}$ and α_j $(n, j = 0, 1, 2, 3, \ldots)$ to be computed for orbital angular momentum l and order of eigenvalue $N(=1, 2, 3, \ldots)$. We thus speak of the Nth eigenvalue $\eta_N(k, l)$, and for this

$$\alpha_0 = \frac{-U_0}{N+l} \tag{2.9}$$

which yields the Coulomb high energy limit

$$\eta_N(k, l) \sim \frac{iU_0}{(N+l)k}. \tag{2.10}$$

The asymptotic expansion for $\eta_N(k, l)$ given in (2.6) is useful only at high energies. However, bearing in mind the high energy behaviour (2.10) and the low energy requirement that $\eta_N(0, l)$ be finite, this expansion can be summed at low energies by the use of an $[n, n-1]$ Padé approximant (Baker 1965)

$$\eta_N(k, l) \simeq \frac{P_{n-1}(K)}{Q_n(K)} = \frac{\Sigma_{m=0}^{n-1} p_m K^m}{\Sigma_{m=0}^{n} q_m K^m} \tag{2.11}$$

($K = ik$) in which it is convenient to adopt the arbitrary normalization

$$q_0 = 1. \tag{2.12}$$

The remaining coefficients of the polynomials in (2.11) are found by matching the right hand side to the first $2n$ terms of the asymptotic expansion for η_N. For example, the $[1, 0]$ approximant is

$$\eta_N(k, l) \simeq \frac{U_0^2}{2U_1(N+l)^2 - U_0(N+l)ik}. \tag{2.13}$$

The poles of the S matrix and T matrix are determined by the condition (Weinberg 1963)

$$\eta_N(k, l) = 1. \tag{2.14}$$

A bound state occurs if this equation has a pure imaginary root

$$k = i\omega \qquad (\omega > 0) \tag{2.15}$$

giving the binding energy ω^2. For a given l there is one bound state for each eigenvalue which is not less than 1 at zero energy. By substituting (2.11) into (2.14) we obtain the polynomial equation

$$\sum_{m=0}^{n-1} (q_m - p_m)K^m + q_n K^n = 0. \tag{2.16}$$

So a bound state at $k = i\omega$ ($\omega > 0$) satisfies

$$\sum_{m=0}^{n-1} (-1)^{m+n}(q_m - p_m)\omega^m + q_n\omega^n = 0. \tag{2.17}$$

Computer calculations have shown that if a given potential $V(r)$ forms a bound state for a particular l and N then convergence to the bound state root occurs very rapidly. The remaining $(n-1)$ roots of equation (2.17) normally lie on the negative imaginary k axis, that is, on the cut in the complex k plane.

For the $[1, 0]$ Padé approximant (2.13) the solution of (2.17) is

$$\omega = \frac{U_0^2 - 2(N+l)^2 U_1}{(N+l)U_0} \tag{2.18}$$

which provides us with a rough check as to whether a local interaction with a nonzero U_0 forms a bound state.

The offshell partial-wave amplitude can be represented by the separable expansion

$$T_l(p_1^2, p_2^2; k^2) = \sum_{N=1}^{\infty} \frac{G_N(p_1, k)G_N(p_2, k)}{(1 - \eta_N(k, l))D_N(k)} \tag{2.19}$$

in which

$$G_N(p,k) = -\int_0^\infty r j_l(pr) V(r) u_l(r)\,\mathrm{d}r$$

$$D_N(k) = -\int_0^\infty V(r) u_l^2(r)\,\mathrm{d}r \tag{2.20}$$

where $j_l(pr)$ is a spherical Bessel function of order l. On the energy shell

$$T_l(k^2, k^2; k^2) = \exp(\mathrm{i}\delta_l)\frac{\sin\delta_l}{k} \tag{2.21}$$

where δ_l is the phase shift with orbital angular momentum l. We have shown that at high energies the functions $G_N(p,k)$ and $D_N(k)$ can be represented by the asymptotic expansions

$$G_N(p,k) = 2(2p)^l l! U_0 K^{-2(l+1)} \sum_{n=0}^{\infty} g_n(p^2) K^{-n} \tag{2.22}$$

and

$$D_N(k) = 2(2l+1)! U_0 (2K)^{-2(l+1)} \sum_{n=0}^{\infty} d_n K^{-n} \tag{2.23}$$

($K = \mathrm{i}k$) where, in general, the coefficients $g_{2n}(p^2)$ and $g_{2n+1}(p^2)$ are polynomials of degree n in p^2. At low energies these expansions can be summed by the use of the $[n+2l+2, n]$ Padé approximants

$$G_N(p,k) \simeq 2(2p)^l l! U_0 \frac{\sum_{m=0}^{n} u_m K^m}{\sum_{m=0}^{n+2l+2} v_m K^m} \tag{2.24}$$

and

$$D_N(k) \simeq 2^{-2l-1}(2l+1)! U_0 \frac{\sum_{m=0}^{n} x_m K^m}{\sum_{m=0}^{n+2l+2} y_m K^m} \tag{2.25}$$

in view of the high energy limits $G_N(p,k) \sim K^{-2(l+1)}$ and $D_N(k) \sim K^{-2(l+1)}$. The polynomial coefficients in (2.24) and (2.25) are determined by the same method as that used to compute the coefficients in (2.11).

In the following sections the results outlined above will be employed to compute Regge poles and the residues at the poles for local interactions $V(r)$ which can be expanded as a power series about $r = 0$, with a $1/r$ singularity at the origin.

3. Polynomial equation for Regge poles

The work summarized in the previous section was carried out for physical values of l ($= 0, 1, 2, 3, \ldots$). We now analytically continue l into the complex orbital angular momentum plane by using equation (2.14) to obtain Regge poles $l(k)$. In this case equation (2.16) becomes a polynomial equation in l whose solution is of the form

$$l(k) = -N + f(k) \qquad [N = 1, 2, 3, \ldots] \tag{3.1}$$

where $f(k)$ is a function of k and of the parameters belonging to the interaction $V(r)$.

For example, the [1, 0] Padé approximant (2.13) yields the quadratic equation

$$2U_1(N+l)^2 - U_0 ik(N+l) - U_0^2 = 0$$

which has the roots

$$l(k) = -N + \left(\frac{U_0}{4U_1}\right)\{ik \pm (8U_1 - k^2)^{1/2}\}. \tag{3.2}$$

At zero energy we have

$$l(0) = -N \pm U_0(2U_1)^{-1/2} \tag{3.3}$$

which gives values that are fairly close to the zero energy results which Lovelace and Masson (1962) calculated for attractive potentials by using high order continued fractions.

According to equation (3.2), branch points occur at

$$k^2 = 8U_1. \tag{3.4}$$

This formula is a good approximation to the branch points obtained by numerical integration of the partial-wave Schrödinger equation (2.1) when $\eta_N(k, l) = 1$ (Warburton 1964 and 1965). At negative energies the values of the Regge poles computed from equation (3.2) are in good agreement with those obtained by Lovelace and Masson (1962) from high order continued fractions. However, at positive energies, the accuracy of the solution (3.2) decreases as the strength of the interaction increases.

In passing, we note that equations (2.10) and (2.14) give the exact Regge pole

$$l(k) = -N - \frac{iA}{2k} \tag{3.5}$$

for the Coulomb potential $V(r) = A/r$.

At high energies the roots (3.2) may be rewritten in the form

$$l(k) = -N + U_0\left\{\frac{K}{4U_1} \pm \left(\frac{K}{4U_1} - \sum_{n=1}^{\infty} \frac{(-4U_1)^{n-1}\Pi_{m=1}^{n}(2m-3)}{n!K^{2n-1}}\right)\right\} \tag{3.6}$$

where $K = ik$. If the minus sign is taken in front of the bracket then this asymptotic expansion agrees to second order in K with that given by Lovelace and Masson.

In principle, when high order Padé approximants are employed in equation (2.16), the latter should lead to results which are more accurate than those obtained from Lovelace and Masson's (1962) continued fraction method. However, when the degree of equation (2.16) is greater than one it is extremely difficult to compute $l(k)$, but on the other hand, it is much easier to use this equation to determine k as a function of l when the latter is restricted to the real axis in the orbital angular momentum plane. Thus, the polynomial equation (2.16) is suitable for tracing Regge trajectories at negative energies. These energies are of interest in the t channel for the scattering amplitude $A(s, t)$ where s and t are the usual Mandelstam variables (Moorhouse 1964, pp 3–12 and pp 229–33). However, the use of Regge poles derived in potential theory to describe such high energy scattering is of doubtful value. For the [1, 0] approximant (2.13) the solution of (2.16) is

$$k = i\left(\frac{U_0}{N+l} - 2(N+l)\frac{U_1}{U_0}\right) \tag{3.7}$$

in which the expression enclosed by the large brackets must be positive for l to be a Regge pole (because, as mentioned in the previous section, the cut in the k plane lies on the negative imaginary axis).

When (3.7) is used to compute the Regge poles of the repulsive Yukawa potential $V(r) = 2\exp(-r)/r$, which has also been studied by Warburton (1964 and 1965) and Abbe (1967), the correct behaviour of each pole is obtained on one side of its branch point only. The incorrect section of each trajectory runs from the branch point to $l = -\infty$. The exact analytic behaviour of the poles should be approached as the degree of equation (2.16) is increased. In principle high order Padé approximants should yield the analytic continuation of the $(2N-1)$th pole into the $2N$th pole, where $N = 1, 2, 3, \ldots$. According to the results presented in tables 1 and 2 it does indeed

Table 1. First and second Regge poles of the potential $V(r) = 2\exp(-r)/r$

l	$-ik$†	l	$-ik$†
−1·20	5·1907	−2·05	1·8708
−1·40	2·8659	−2·06	1·8716
−1·60	2·1984	−2·07	1·8728
−1·80	1·9461	−2·08	1·8742
−2·00	1·8708	−2·09	1·8759
−2·01	1·8703	−2·10	1·8778
−2·02	1·8700	−2·20	1·9158
−2·03	1·8700	−2·30	2·0022
−2·04	1·8703	−2·40	2·1797

† Degree of equation (2.16) = 10 for $\eta_1(k, l) = 1$

Table 2. Second Regge pole of the potential $V(r) = 2\exp(-r)/r$

l	$-ik$†	l	$-ik$†
−2·01	100·000	−2·09	11·150
−2·02	50·010	−2·10	10·042
−2·03	33·348	−2·15	6·722
−2·04	25·019	−2·20	5·063
−2·05	20·023	−2·25	4·061
−2·06	16·694	−2·30	3·376, 1·889
−2·07	14·317	−2·35	2·844, 2·063
−2·08	12·535	−2·40	2·067

† Degree of equation (2.16) = 10 for $\eta_2(k, l) = 1$.

appear that the first Regge pole has been continued into the second pole by solving the polynomial equation for $\eta_1(k, l) = 1$. We note from table 1 that the branch point connecting these poles occurs at $l \simeq -2{\cdot}03$ with $k = 1{\cdot}87\mathrm{i}$ which is in excellent agreement with Warburton's (1964) calculation. It can be seen from table 2 that this branch point was not located when equation (2.16) was solved for $\eta_2(k, l) = 1$. The rate of convergence of the polynomial equation is displayed in table 3.

It therefore seems that the polynomial equation (2.16), provided it is of sufficiently high degree, can be employed to determine Regge poles, possessing correct analytic properties, for local interactions which can be expanded as a power series about $r = 0$

Table 3. Rate of convergence of equation (2.16) for the first and second Regge poles of the potential $V(r) = 2\exp(-r)/r$

l	$-ik$					
	$n = 2$	$n = 4$	$n = 6$	$n = 8$	$n = 9$	$n = 10$
−1·75	1·9869	1·9879	1·9880	1·9880	1·9880	1·9880
−2·30	1·9124	1·9611	1·9974	2·0011	2·0022	2·0022

n = degree of equation (2.16) for $\eta_1(k, l) = 1$.

with a $1/r$ singularity at the origin. The equation can be solved extremely rapidly on a computer. As the degree of the equation is increased from unity to (for example) 10 it is found that physical roots (ie the roots yielding Regge poles) are stable whilst the unphysical roots are generally unstable, the locations and movements of the latter normally occurring only on the cut in the complex k plane.

4. The residue at a Regge pole

The residue β at the Regge pole l_0 is defined by (Ahmadzadeh 1963)

$$\beta = \lim_{l \to l_0} (l - l_0) S(k, l) \tag{4.1}$$

where $k = k_0$ at $l = l_0$. $S(k, l)$ is the partial-wave S matrix. Weinberg (1963) has shown that

$$S(k, l) = \prod_{N=1}^{\infty} \left(\frac{1 - \eta_N(-k, l)}{1 - \eta_N(k, l)} \right) \tag{4.2}$$

at both positive and negative energies.

As $\eta_N(k_0, l_0) = 1$ we can expand $\eta_N(k, l)$ about (k_0, l_0) as the Taylor series

$$\eta_N(k, l) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(k - k_0)^m (l - l_0)^n}{(m+n)!} \left(\frac{\partial^{m+n} \eta_N}{\partial k^m \partial l^n} \right)_{(k_0, l_0)} \tag{4.3}$$

which yields

$$\eta_N(k_0, l) \simeq 1 + (l - l_0) \left(\frac{\partial \eta_N}{\partial l} \right)_{(k_0, l_0)} \tag{4.4}$$

if we assume that the terms containing higher derivatives can be ignored. If the Mth eigenvalue is responsible for the pole, then by substituting (4.2) and (4.4) into (4.1) we find that

$$\beta_M \simeq - \frac{1 - \eta_M(-k_0, l_0)}{(\partial \eta_M / \partial l)_{(k_0, l_0)}} \prod_{\substack{N \\ N \neq M}} \left(\frac{1 - \eta_N(-k_0, l_0)}{1 - \eta_N(k_0, l_0)} \right). \tag{4.5}$$

When k_0 is situated on the positive imaginary axis in the k plane its value must be such that $-k_0$ will be above the cut on the negative imaginary axis to enable this result to be employed to calculate the residue. This restriction imposes an upper limit on the strength A of the attractive Yukawa potential $V(r) = -A\exp(-\mu r)/r$ since the cut occupies the region $\operatorname{Im} k \leqslant -\mu/2$.

We shall now derive another formula for β which will not involve evaluating $\eta_N(-k_0, l_0)$. As $S(k, l) = \exp(2i\delta_l)$, equations (2.19) and (2.21) allow the S matrix to be expressed in the form

$$S(k, l) = 1 + 2ik \sum_{N=1}^{\infty} \frac{G_N^2(k, k)}{(1 - \eta_N(k, l))D_N(k)} \tag{4.6}$$

If the Nth eigenvalue is responsible for the Regge pole, equations (4.1), (4.4), and (4.6) lead to the result

$$\beta_N \simeq -\frac{2ik_0 G_N^2(k_0, k_0)}{D_N(k_0)(\hat{c}\eta_N/\hat{c}l)_{(k_0, l_0)}} \tag{4.7}$$

where $G_N(k_0, k_0)$ and $D_N(k_0)$ are evaluated at $l = l_0$. When this simple formula is applied to the attractive Yukawa potential $V(r) = -5\exp(-r)/r$, which forms an S wave bound state at $k_0 = 1{\cdot}5747i$, we find that the residue at the bound state pole is $\beta_1 = -5{\cdot}445$ which is in good agreement with the result obtained by Ahmadzadeh (1963).

We end this section with an investigation of the high energy behaviour of β. The expansions (2.22) and (2.23) yield the offshell high energy limit

$$\frac{G_N^2(p, k)}{D_N(k)} \sim \frac{2^{(4l-3)}(l!)^2 U_0 p^{2l} K^{-2(l+1)}}{(2l+1)!} \tag{4.8}$$

which reduces to

$$\frac{G_N^2(k, k)}{D_N(k)} \sim \frac{2^{(4l+3)}(-1)^{l-1}(l!)^2 U_0}{(2l+1)!k^2} \tag{4.9}$$

on the energy shell. The Coulomb high energy limit (2.10) gives

$$\frac{\hat{c}\eta_N}{\hat{c}l} \sim \frac{-iU_0}{(l+N)^2 k}. \tag{4.10}$$

By substituting (4.9) and (4.10) into (4.7) we obtain the approximation

$$\beta_N \sim \frac{(-16)^{l+1}(l!)^2(l+N)^2}{(2l+1)!} \tag{4.11}$$

and by employing the Coulomb high energy behaviour

$$l(k) \sim -N + \frac{iU_0}{k} \tag{4.12}$$

we find that

$$\beta_1 \sim (-16)^{iU_0/k}\left(\frac{iU_0}{k}\right)^2 \frac{(\Gamma(iU_0/k))^2}{\Gamma(2iU_0/k)} \tag{4.13}$$

where $\Gamma(z) = (z-1)!$. As $\Gamma(1/z) = z\Gamma(1+1/z) \sim z$ when $|z| \to \infty$ it is easily seen that the high energy limit for the residue is

$$\beta_1 \sim \frac{2iU_0}{k} \tag{4.14}$$

which agrees exactly with the formula for the Coulomb potential derived by Ahmadzadeh (1963).

5. Conclusions

It has been shown that the polynomial equation (2.16), which can be solved very rapidly on a computer, is particularly suitable for determining Regge poles at negative energies for local interactions with a $1/r$ singularity at the origin. The residue at any pole can then be computed from the simple formula (4.7). Approximate values of Regge poles at positive or negative energies can be calculated very easily from equation (3.2). An estimate of the scattering amplitude $A(s, t)$ can be made from a knowledge of Regge poles and the residues at these poles (Moorhouse 1964, pp 223–58, Aly and Narayanaswamy 1969).

It is well known that, for the class of potentials considered in this paper, a plot of Re l against energy yields a trajectory which initially rises to a maximum value and then falls as the energy increases. However, high energy physics experiments indicate that Regge trajectories rise continuously with increasing energy (Mandelstam 1969, Squires 1971). Aly and Narayanaswamy (1969) have shown that potentials with a $1/r^4$ behaviour yield continuously rising trajectories. It should be possible to study Regge poles of local interactions with a $1/r^n$ behaviour, where $n > 1$, by employing the finite difference solution of the partial-wave Schrödinger equation (2.1) developed in another paper (Stern and Warburton 1972). The finite difference method can deal with any local potential but is less efficient than the method described in the preceding sections of this paper when calculations are performed with interactions possessing a $1/r$ singularity at the origin. This latter class of central potentials (expressed in a generalized form by the equations dealing with $V(r)$ and $U(r)$ in (2.3) and (2.6)) is frequently employed to represent interactions in low energy problems (Coester and Yen 1963, Mongan 1969).

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